Question	Score
1	/10
2	/16
3	/24
4	/25
5	/25
Total	/100

Applied Algorithms, Final exam

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Good Luck!

Question 1 (10 pts.) :

An instance of n=10 jobs is assigned on two identical machines in a way that minimizes the total completion time (optimal solution for $P_2||\Sigma C_j$.).

The processing time of each of the 10 jobs is increased by one.

Is it possible to know by how much the value of the optimal solution increases? If yes, specify this value. If not, explain why.

In an optimal SPT schedule, there are 5 jobs on each machine. The completion time of the i-th job is increased by i. The total completion time on both machines, is increased by 2(1+2+3+4+5)=30.

Question 2 (16 pts.):

Let I={0.8, 0.6, 0.3,0.2, 0.1} be the set of item sizes in an instance of Bin-Packing.

1. (5 pts.) What is the output of **Next-Fit** algorithm, assuming the items are considered from largest to smallest? Present the packing.

Bin 1: {0.8}

Bin 2: {0.6, 0.3}

Bin 3: {0.2. 0.1}

2. (5 pts.) What is the output of **First-Fit Decreasing** algorithm? Present the packing.

Bin 1: {0.8, 0.2,}

Bin 2: {0.6, 0.3, 0.1}

3. (6 pts.) Suggest an order according to which the items are considered, such that **First-Fit** algorithm is not optimal for this order. Write the order and present the packing.

There are several possible correct answers. Here is one option:

0.8, 0.1, 0.6, 0.2, 0.3

Bin 1: {0.8, 0.1}

Bin 2: {0.6, 0.2}

Bin 3: {0.3}

Question 3 (3*8 =24 pts.)

For each of the following three claims, determine whether it is true of false. If true, justify shortly (no need to prove formally). If false, give a counter example.

Let G be an undirected path with an odd number of vertices and arbitrary edge weights.

1. The middle node is an optimal **vertex 1-center**.

False. Consider a 5-node path a-b-c-d-e with edge lengths a-1-b-1-c-1-d-10-e. If the center is placed on c (the middle node) then the maximal distance is 11 (from e). If the center is placed on d then the maximal distance is 10 (from e), so c is not an optimal solution.

2. The optimal **absolute 1-center** is identical to the optimal **vertex 1-center**.

False. Consider a 3-node path with different edge lengths, for example a-1-b-3-c.

The absolute 1-center is located on the longer edge. The vertex 1-center is located in the middle node.

3. Assume nodes have **unit weights**. The middle node is an optimal **1-median**.

Note: this is a complete formal proof. To get full credit it was sufficient to only specify the exchange argument without proving it.

Let the number of nodes in the lace be 2k+1. Let A be the middle node, Assume that the optimal solution is not on A but on a node B that is left (without loss of generality) of A. Let d denote the distance between A and B. Consider the solution in which the facility is located on A. Compare the value of this solution to the solution in which the facility is on B.

For the k vertices that are right of A, their distance to the facility is decreased by d (it is now closer for them). Similarly, for A the distance is decreased by d (from d to 0).

For the vertices that are left of B and for B itself the distance to the facility is increased by d. Assume there are j such vertices (including B). For the k-j nodes between A and B the distance might increase or decrease, we cannot tell, but it cannot increase by more than d. In total, for k+1 vertices the distance decreases by d, for j vertices the distance increases by d and for k-j vertices, the distance might increase by at most d. Altogether the total distance is decreased by at least $(k+1)d-kd=d > 0$, contradicting the assumption that locating the facility on B is optimal.

Question 4 (25 points):

Consider the problem $1|r_j,p_j=1|L_{max}$. In this problem, all the jobs have unit processing times (that is, for all j, $p_j=1$), every job $1 \le j \le n$ is associated with release time r_j, and due-date d_j. The jobs should be assigned on a single machine in a way that minimizes the maximal lateness $L_{max} = \max_j L_j$, where $L_j=C_j-d_j$.

1. Suggest an optimal algorithm of the problem.
2. Prove that your algorithm is optimal.

Algorithm: at any time, schedule an available job with the smallest due date.

Proof: The proof uses an exchange argument. Assume that an optimal schedule S violates the algorithm. Specifically, let i,k be two jobs such that both are available at time t ($r_i \le t$, $r_k \le t$), $d_i > d_k$ but job i precedes job k in S.

Consider the schedule S' obtained from S by swapping jobs i and k.

$L'_j=L_j$ for any job $j \ne i,k$

$L'_i=C'_i-d_i=C_k-d_i < C_k-d_k=L_k$

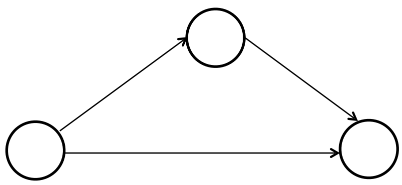
$L'_k=C'_k-d_k=C_i-d_k<C_k-d_k=L_k$

Therefore $L_{max}(S') \le L_{max}(S)$.

If $L_{max}(S')<L_{max}(S)$ then we have obtained a contradiction to the assumption that S is optimal.

If $L_{max}(S')=L_{max}(S)$ then S can be modified without increasing the objective function value such that jobs i,k do not violate the algorithm.

Question 5 (25 pts.):



Consider the following **network congestion game**.

All edges have the same linear cost function $c_j(k)=k$. Specifically, $c_{sa}(k)= k$, $c_{at}(k)=k$, and $c_{st}(k)=k$.

Assume that 14 players need to use the network.

4 players need a path from s to a, and 10 players need a path from s to t. Formally, for 4 players the only strategy is {sa} and for 10 players there are two strategies: {sa, at}, {st}.

1. Complete the following computation of a Nash Equilibrium profile in this game:

A profile is characterized by the number of (s-t)-players that choose the upper path.

Denote this number x. $0 \leq x \leq 10$.

For a given x, the cost of the strategy {sa, at} is ___4+x+x________, and the cost of the strategy {st} is __10-x_______. The strategy is a NE when ______4+x+x=10-x_____, that is, for x=_2_______.

The NE profile is therefore: 4 players are using their only possible path {sa}, each having cost ___6____ , ___2___ players are using {sa, at}, each having cost ___8___, and ___8____ players are using the lower path, each has cost ____8______.

2. What is the social optimum for this game (with respect to the objective of minimizing the total players' cost)? Explain your solution.

The total cost for a given x is $4(4+x)+x(4+2x)+(10-x)(10-x)= 3x^2-12x+116$.

The minimal value is achieved when (derivation) $6x=12$, that is, $x=2$.

The social optimum value is $3\cdot2^2-12\cdot2+116 = 104$.

3. What is the price of Stability in this game? Explain your solution.

PoS=1 because the optimal solution is a NE.